A Vector Surface Integral Approach to Computing Inductances of General 3-D Structures

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Abstract

A new approach to calculating the magneto-quasi-static but frequency-dependent inductance of three-dimensional conductors is presented. A vector surface integral formulation is used, as this requires only a conductor surface discretization, and an excitation source which ensures current conservation is self-consistently computed. Results are presented to demonstrate that the method is effective for computing the inductance and the resistance for general 3-D structures.

1 Introduction

Computing the correct inductance and resistance values for arbitrary three-dimensional conductor geometries is important to designers of the interconnect in high-speed digital systems. Currenty, three-dimensional inductance calculation techniques are based on the volume integral method [1] [2] [3]. These methods require that the conductor volume be discretized, and to correctly model the skin effect at high-frequency a very fine discretization of the conductor volume near the surface is needed. Therefore, when using volume integral methods, it is common to use different discretizations for different frequency ranges.

In this short paper, we present a new surface integral method which avoids volume discretization. The basic idea is similar to the approaches used by [4] [5] [6] for two-dimensional inductance calculations. In these other efforts, the problem is formulated as a surface integral equation using Green's functions, and then solved using a boundary element method. In two-dimensions, however, the surface formulation is simpler, because the unknowns are scalar and an excitation source is easily derived. In three-dimensions, the unknowns are vector quantities on conductor surfaces, and an excitation source which ensures current conservation must be computed.

In Section 2, the basic surface formulation under the magneto-quasi-static approximation is described. Also given is the approach to the excitation calculation and expressions for the inductance and resistance in terms of the vector unknowns on the surfaces. The discretization scheme for the vector surface equations as well as the methods for evaluating integrals of the Green's functions are explained in Section 3. In Section 4, the results for an example conductor structure are presented and compared to the previous work. Finally, conclusions and acknowledgements are given in Section 5.

2 Formulation

Under the magneto-quasi-static assumption [7], the Maxwell's equations for the region inside the conductor are,

$$\nabla \times \vec{H}(\vec{r}) = \sigma \vec{E}(\vec{r}) = \sigma(i\omega \vec{A}(\vec{r}) - \vec{\nabla}\phi(\vec{r})) \quad (1)$$

$$\nabla \times \vec{E}(\vec{r}) = \nabla \times (i\omega \vec{A}(\vec{r}) - \vec{\nabla}\phi(\vec{r})) = i\omega\mu \vec{H}(\vec{r}), \quad (2)$$

and for the outside dielctric region are,

$$\nabla \times \vec{H}(\vec{r}) = 0 \quad (3)$$

$$\nabla \times \vec{E}(\vec{r}) = \nabla \times i\omega \vec{A}(\vec{r}) = i\omega\mu \vec{H}(\vec{r}), \quad (4)$$

By using the dyadic Green's function, the Maxwell's equations can be transformed into the following vector surface equations in terms of the tangential electric and magnetic fields on the conductor

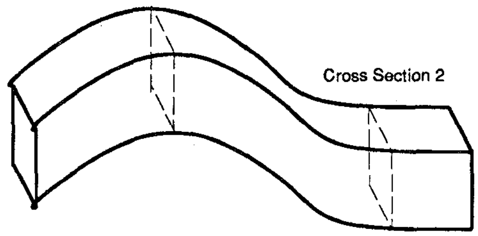

Figure 1: Current conservation condition imposed at different cross sections

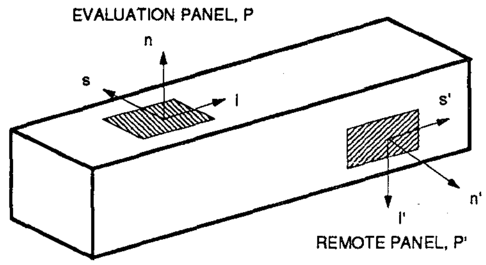

Figure 2: Local coordinate systems for panels

surfaces [8]. There are two sets of such integral equations, one for outside the conductor,

$$-\frac{1}{2}i\omega\vec{A}(\vec{r}) + \int_s \{(\vec{n}' \times i\omega\vec{A}(\vec{r}')) \times \nabla g_{out}(\vec{r},\vec{r}')da'$$
$$+ \int_s (\vec{n}' \times i\omega\mu\vec{H}(\vec{r}'))g_{out}(\vec{r},\vec{r}')\}da' = 0, \qquad (5)$$

and the other for inside the conductors,

$$\frac{1}{2}\vec{H}(\vec{r}) + \int_s \{(\vec{n}' \times \vec{H}(\vec{r}')) \times \nabla g_{in}(\vec{r},\vec{r}')\}da'$$
$$+ \int_s \{(\vec{n}' \cdot \vec{H}(\vec{r}'))\nabla g_{in}(\vec{r},\vec{r}')\}da'$$
$$+ \int_s \{\sigma(\vec{n}' \times i\omega\vec{A}(\vec{r}'))g_{in}(\vec{r},\vec{r}')\}da'$$
$$= \int_s \{\sigma(\vec{n}' \times \vec{M}_s(\vec{r}'))g_{in}(\vec{r},\vec{r}')\}da'. \qquad (6)$$

Here, the Green's functions are $g_{out}(\vec{r},\vec{r}') = \frac{1}{4\pi\|\vec{r}-\vec{r}'\|}$, and $g_{in}(\vec{r},\vec{r}') = \frac{e^{-\|\vec{r}-\vec{r}'\|/\delta_{skin}}e^{i\|\vec{r}-\vec{r}'\|/\delta_{skin}}}{4\pi\|\vec{r}-\vec{r}'\|}$, where $\delta_{skin} = \sqrt{\frac{2}{\omega\mu\sigma}}$, the skin depth at frequency ω. These Green's functions are referred to as the free-space Green's and the spherical Hankel's functions, respectively.

Note that under the magneto-quasi-static assumption, the surface integral equations are written in terms of the vector potential, $i\omega\vec{A}(\vec{r})$, and the magnetic field, $\vec{H}(\vec{r})$. The electric field due to the scalar potential, $\vec{\nabla}\phi(\vec{r})$, appears as the magnetic surface current, $\vec{n} \times \vec{M}_s(\vec{r})$, which is used to generate volume current in the conductor [6]. The direction of the magnetic current is along the perimeter of cross section cuts of the conductor.

The two equations are solved for the tangential vector potential and the magnetic field on the conductor surfaces, which are continuous across the boundary between the conductor and the dielectric. The magnitudes of the magnetic surface current, $\vec{n} \times \vec{M}_s(\vec{r})$ are determined to enforce current conservation condition. That is, the volume current through any cross sections for a single conductor is constant. For example, in Figure (1) the total current through the cross sectons 1 and 2 are the same, and can be written in terms of the line integrals of the magnetic field as,

$$\oint_{sec\ 1} \vec{H}(\vec{r}) \cdot \vec{dl} = \oint_{sec\ 2} \vec{H}(\vec{r}) \cdot \vec{dl} = constant.$$

Here, the line integrals are taken over the closed boundaries of the cross sections of the conductor at different locations. Finally, the expressions for the inductance and resistance in terms of the surface field quantities can be derived from an energy argument as,

$$L = -\frac{1}{\omega}\mathrm{IM}(\int_{surf} \vec{H}(\vec{r})^* \cdot (\vec{n} \times \vec{M}_s(\vec{r}))da)/|current|^2$$

$$R = \mathrm{RE}(\int_{surf} \vec{H}(\vec{r})^* \cdot (\vec{n} \times \vec{M}_s(\vec{r}))da)/|current|^2$$

where, $current = \oint_{ring} \vec{H}(\vec{r}) \cdot \vec{dl}$.

3 Discretization

The conductor surfaces are discretized into flat quadrilateral panels on which the magnetic and electric field quantities are assumed constant. Since the surface integral equations are in vector form, it is necessary to assign local coordinate system, $\vec{l} \times \vec{s} = \vec{n}$, to each panel as shown in Figure (2). The discretized version of the integral equation (5) is, for example,

$$-\frac{1}{2}i\omega A_l(\vec{r}) + \Sigma_{all\ p'}[i\omega A_l(\vec{r}')(\vec{l}\cdot\vec{l}' \int_{p'} \frac{\partial}{\partial n'}g_{out}(\vec{r},\vec{r}')da'$$
$$-\vec{l}\cdot\vec{n}' \int_{all\ p'} \frac{\partial}{\partial l'}g_{in}(\vec{r},\vec{r}')da') + i\omega A_s(\vec{r}')$$

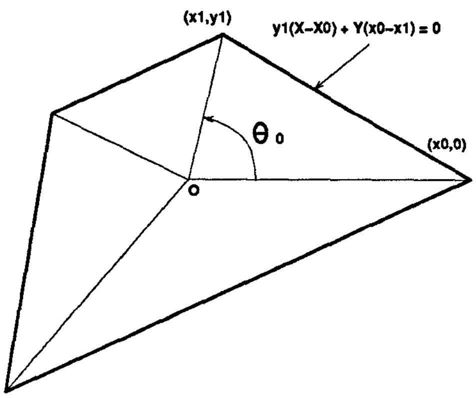

Figure 3: Non-singular Method

$$(\vec{l}\cdot\vec{s}'\int_{p'}\frac{\partial}{\partial n'}g_{out}(\vec{r},\vec{r}')da' - \vec{l}\cdot\vec{n}'\int_{p'}\frac{\partial}{\partial s'}g_{out}(\vec{r},\vec{r}')da')]$$

$$+\Sigma_{all\ p'\ \&\ p}[(\vec{l}\cdot\vec{s}'i\omega\mu H_l(\vec{r}') - \vec{l}\cdot\vec{l}'i\omega\mu H_s(\vec{r}'))$$

$$\int_{p'} g_{out}(\vec{r},\vec{r}')da'] = 0 \qquad (7)$$

Equation (7) simply relates $\vec{l}$ component of the vector potential at a given panel, $i\omega A_l(\vec{r})$, to the tangential field quantities on all the panels representing the conductor surfaces.

To integrate the free-space Green's function, $g_{out}(\vec{r},\vec{r}') = \frac{1}{4\pi\|\vec{r}-\vec{r}'\|}$, over an arbitrary quadrilateral panel, a closed form expression is available [9]. As for the spherical Hankel's function, $g_{in}(\vec{r},\vec{r}') = \frac{e^{-\|\vec{r}-\vec{r}'\|/\delta_{skin}}e^{i\|\vec{r}-\vec{r}'\|/\delta_{skin}}}{4\pi\|\vec{r}-\vec{r}'\|}$, there is no such formula. If $\vec{r}$ is not on the panel, this integration can be performed easily numerically. For the case when $\vec{r}$ is on the panel, the integrand has an integrable singularity which must be removed. To accomplish this, consider subdividing a panel as shown in Figure (3), and rewriting the integral of the Green's function in polar coordinates. Then the integral of the spherical Hankel's function over the region shown in Figure (3) becomes,

$$\int_0^{\theta_0}\{\exp(\frac{-x0/\delta_{skin}}{\cos(\theta)+\sin(\theta)(x0-x1)/y1})$$

$$\exp(\frac{i\ x0/\delta_{skin}}{\cos(\theta)+\sin(\theta)(x0-x1)/y1})\}d\theta,$$

which has no singularity. The integrals involving partial derivatives of the Green's functions can be evaluated by finite differences.

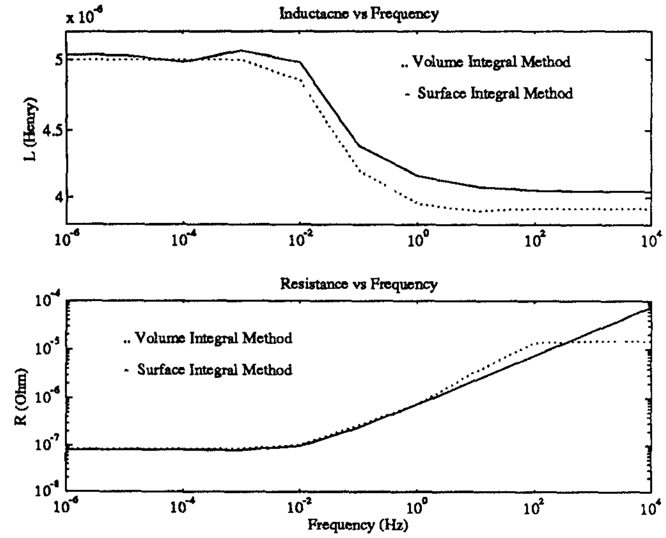

Figure 4: Results for Two Parallel Rectangular Conductors

4 Results

The inductance and resistance for the two parallel rectangular conductors were computed using the above technique. The conductors' dimensions are 2 by 2 by 10 meters, and they are separated by 2 meters. The problem is treated as a single loop, with one conductor acting as the return path. The results are shown in Figure 4 along with those from FASTHENRY which uses the volume integral method [1]. Each conductor's surface was discretized into 120 panels for the surface integral technique, while each conductor's cross-section was discretized into 121 filaments for FASTHENRY. Results for more interesting conductor structures shown in Figures (5) and (6) are reported in Figure (7).

5 Conclusion

A new three-dimensional inductance calculation technique that can handle arbitrary conductor geometries was implemented and compared to a standard volume-element method. As shown in Figure (4) the results are in excellent agreement for the medium to high frequency range. In particular, the resistance at very high frequencies can be correctly modelled using the new surface integral method. The results for the two other examples shown in Figures (6) and (5) demonstrate this surface integral approach's ability to

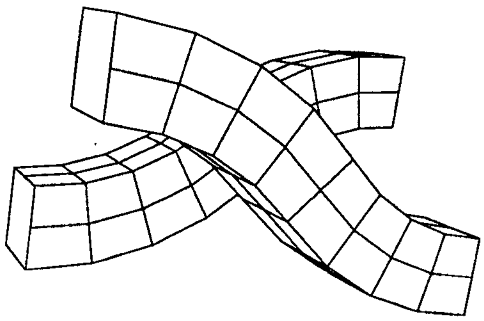

Figure 5: Two Curved Conductors

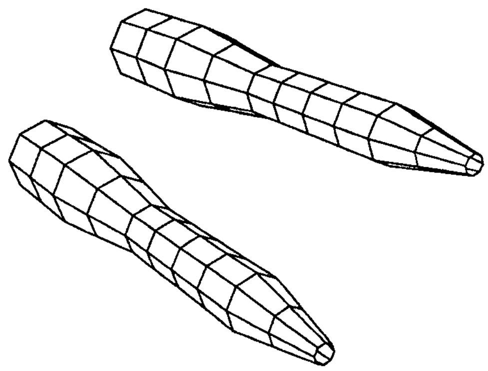

Figure 6: Two Adjacent Pin Structures

compute the inductances and the resistances of conductors with curved surfaces.

For low frequencies, however, the inductances calculated using this surface integral method have errors, as can be seen in Figure (7). This problem is currently being investigated, and the results from a full numerical study of the new technique will be reported in future studies.

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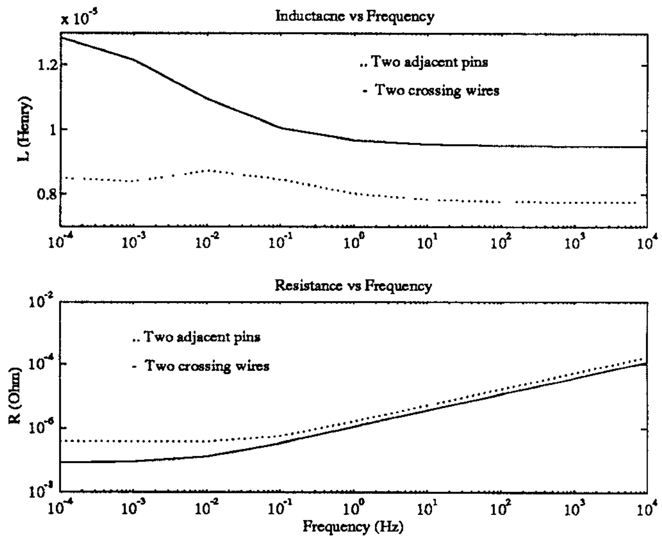

Figure 7: Results for Two Curved Conductors and Two Pin Structures

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